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LAMBDA RATIO

Surface roughness must be accurately characterized in order for the lambda ratio to be a completely effective tool.

THE LAMBDA RATIO WAS ORIGINALLY DEVELOPED TO quantify the quality of lubricant operating regimes relative to bearing performance [1]. Starting around the 1990s, the lambda ratio has been tested and used to define optimal lubricant regimes for gears. In both cases, the lambda ratio has been found to correlate reasonably well with surface contact fatigue [2, 3]. As a result, the lambda ratio is commonly used in component and system design, and, as such, its accuracy is critical to ensuring that the predicted performance levels are achieved.

LAMBDA: BRIEF HISTORY AND BASIC CALCULATION

The tribology concept of the lambda ratio (λ) — the ratio of lubricant film thickness (h) to composite surface roughness (σ) — has existed for over 50 years. While the equation shown in Figure 1 appears simple on first review, it is, in fact, exceptionally complicated and multifaceted. Both areas of influence in the lambda ratio (film thickness and composite surface roughness) have garnered considerable study over the years. The most complex and investigated aspect of the lambda ratio is arguably the lubricant film thickness. Lubricant film thickness is affected by a multitude of different factors including viscosity, temperature, relative surface velocity, load, contact area, component plastic deformation, and lubricant compressibility. The dynamic nature of power transfer systems like gears and bearings makes the calculation of lubricant film thickness complex. As a result, there have been many different equations proposed and used over the years.

Figure 1: Lambda ratio

$$\lambda = \frac{h}{\sigma}$$

CALCULATION OF LUBRICANT FILM THICKNESS

The first proposed solution to lubricant film thickness as applied to line-contact EHL (elasto-hydrodynamic lubrication) is credited to Grubin and Ertel [4]. Their method seeks to define central film thickness while making the assumptions that line contact is infinite, lubricants follow Newtonian behaviors, and fluids are incompressible. The Ertel-Grubin method has been utilized in numerous technical studies since its creation, and its value is still discussed even today. However, it is also widely recognized that Ertel and Grubin's solution does not take into account many potential factors in calculating film thickness. Additional one-dimensional equations were developed to take more of these variables into account including the Dowson and Higginson equation shown in Figure 2. This equation estimates the minimum film thickness on the exit side of contact [5]. However, the Dowson-Higginson equation and other one-dimensional film thickness equations are still limited in that they do not allow for the consideration of an elliptical (i.e., two-dimensional) contact zone. As one researches the myriad of film thickness calculations, the aforementioned complexity becomes clear. Study of these film thickness calculations, while valuable and warranted, is only half of the story.

$$\frac{Wh_{min}}{\eta_0 URB} = 2.65\left(\frac{W^{3/2}\alpha}{\eta_0 U^{1/2}R}\right)^{0.54}\left(\frac{W}{B(URE)^{1/2}}\right)^{0.06}$$

Figure 2: Dowson-Higginson solution [6]

CALCULATION OF COMPOSITE SURFACE ROUGHNESS

The other half of the story is composite surface roughness. Composite surface roughness appears simple and straightforward, and in some ways, it has been treated as the less complex element of the lambda ratio. In general, there are two accepted formulas for calculating composite surface roughness: one that uses Rq and one that uses Ra. Other formulas exist that take into account contact width and other variables, but these formulas are still fundamentally based on Ra or Rq. Both Ra and Rq are two-dimensional surface profile evaluations that are based on measuring the peaks and valleys of a surface and establishing a mean line between those peaks and valleys. Both are dependent on the evaluation length and a filtering factor meant to account for waviness in the surface. Ra is the arithmetic average of the absolute values of the profile heights over the evaluation length, and Rq is the root mean square average of the profile heights. The primary difference between Ra and Rq is that Rq tends to amplify outlier profile measurements.

Figure 3: Composite roughness equations

$$\sigma_q = \sqrt{{R_{q1}}^2 + {R_{q2}}^2}$$

$$\sigma_a = R_{a1} + R_{a2}$$

Figure 4: Ra and Rq equations

$$R_a = \left(\frac{1}{L}\right)\int_0^L |Z(x)|dx$$

$$R_q = \sqrt{\left[\left(\frac{1}{L}\right)\int_0^L Z(x)^2 dx\right]}$$

ISSUES WITH RA AND RQ

Ra and Rq have at least two primary issues relative to their accuracy in calculating composite surface roughness. The first issue is an obvious one: they are both two-dimensional surface evaluations. With either Ra or Rq, surface roughness is being classified via the use of a single line on the surface of a component. Clearly, the use of a single line to evaluate the quality of a surface and the expectation that the surface roughness will be significantly similar throughout the contact area is a considerable leap of faith given the variability of most mechanical forming or finishing operations. It is worth noting that isotropic superfinishing processes will significantly reduce this "line-to-line" variation across the applicable surface.

The second issue is that both Ra and Rq provide equal weighting to the respective peaks and valleys on the component's surface. As a result, these measurements can become potentially misleading when used to determine the lambda ratio if the relative peak height and valley depth (from the mean line) is not both consistent and relatively sinusoidal. With a machined surface, it is the peak asperities that will penetrate through the lubricant film should operating conditions deviate from expectations. Hence, one could argue that these peaks should be given greater weighting in the composite roughness calculation. However, with a partially planarized surface (such as from honing or polish-grinding) where residual deep valleys will exist, there can be problems with localized pressure differentials creating rapid reductions in film thickness. These reductions can lead to surface contact or other surface fatigue initiating events. In this scenario, the valley depth seems to be the critical variable. So, depending upon the final forming step and how representative the roughness measurement is of the surface at large, the Ra/Rq measurements may understate the effective composite roughness and derivatively overstate the lambda ratio.

CONCLUSION

Historic data has proven that the lambda ratio is a valuable method of predicting safe operating conditions for lubricated power transfer systems. While the calculation of the lubricant film thickness undoubtedly justifies a great deal of attention and study, it is also worthwhile to study alternate methods of quantifying the composite surface roughness. Ultimately, it may be that a single roughness measurement parameter is inadequate to fully classify a surface. Given the value that is placed on the lambda ratio relative to component and system design, it is important to ensure that both the lubricant film thickness and the composite surface roughness are quantified as accurately as possible.

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